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Excited Q-Balls in the MSSM with gravity mediated supersymmetry breaking

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Abstract

Excited Q-balls are studied by numerical simulations in the Minimal Supersymmetric Standard Model with supersymmetry broken by a gravity mediated mechanism. It is found that there is a suppression factor of $\mathcal{O}(10^{-2})$ in the rate at which a Q-ball can emit their excess energy compared to the rate set by the dynamical scale of the field, m . Furthermore, it is noted that a Q-ball can withstand a large amount of excess energy without losing a significant amount of its charge. The cosmological importance of these properties are considered for Q-balls in the thermal bath of the early universe.

1 Introduction

A scalar field theory with a global $U(1)$ symmetry can contain a class of non-topological solitons, Q-balls [1]. A Q-ball is the ground state of the theory in the sector of fixed charge so that its stability and existence are due to the conservation of the $U(1)$ charge. In realistic theories, Q-balls are generally present in supersymmetric extensions of the Standard Model [2]. In particular the scalar potential of the Minimal Supersymmetric Standard Model (MSSM) has several flat directions [3] that can support lepton and/or baryon number carrying Q-ball solutions in its spectrum.

In cosmology, Q-balls can play a significant role in the phenomenology of the early universe. The Affleck-Dine condensate is unstable with respect to fragmentation into Q-balls so that the small perturbations naturally present in the AD-condensate lead to the formation of Q-balls [4, 5]. The fate of the Q-balls then depends on the supersymmetry (SUSY) breaking mechanism: if SUSY is broken by a gauge mediated mechanism they can be absolutely stable and contribute to the dark matter content of the universe [6], whereas if the SUSY breaking mechanism is gravity mediated, the Q-balls will be short-lived

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but can still have interesting cosmological consequences, *e.g.* they may explain the ratio of baryons to dark matter in the universe [7]. The charge spectrum and the number of balls is dependent on the form of the potential and initial conditions of the AD-condensate [8, 9, 10].

After the Q-balls have formed, they will be subjected to the thermal bath of the early universe. The effect of the background clearly depends on how energetic the background radiation is and how strongly the Q-ball field couples to it. The temperature of the universe is hence significant in determining the importance of the thermal background for Q-balls.

Thermal effects on Q-balls have been previously considered via two different mechanisms, dissociation [11, 5] and dissolution [11]. Dissociation of Q-balls proceeds through the collisions of thermal particles with the “hard-core” of a Q-ball so that excess energy is transferred to the Q-ball. This excess energy can then overcome the binding energy and dissociate the Q-ball. Dissolution occurs due to the thermalization of the “soft edge” of the Q-ball leading to charge transport from the Q-ball to the background. In this work, dissolution and dissociation have been considered in the gravity mediated scenario by studying the time development of Q-balls with initial excess charge. The gauge mediated case has been studied by thermodynamical means in [12].

In addition to thermal erosion, Q-balls are also subjected to evaporation [13], where Q-balls lose their charge via pair production at their surface. The loss of charge through evaporation is generally much slower than that due to thermal processes so evaporation becomes effective only after the universe has cooled enough for the thermal effects to be insignificant.

The scalar potential in the case of MSSM with SUSY broken by a gravity mediated scenario is given by (omitting the A -terms)[11]

$$V(\Phi) = m^2 |\Phi|^2 (1 + K \log(\frac{|\Phi|^2}{M^2})) + \frac{\lambda^2}{M_{Pl}^{2(d-3)}} |\Phi|^{2(d-1)}, \quad (1)$$

where d is the dimension of the non-renormalizable term in the superpotential, m is the mass scale given by the SUSY breaking scale, M is a large mass scale and K a negative constant of order one. The interesting cases for Q-balls are the $d=4$ $u^c u^c d^c e^c$ and $d=6$ $u^c d^c d^c$ directions in the superpotential. Q-balls in this potential are well represented by a Gaussian approximation [11], where the Q-ball field is approximated by $\phi(t, \vec{x}) = e^{i\omega t} \phi_0 e^{-r^2/R^2}$. Here ω is the so called Q-ball frequency and $R \sim m^{-1} |K|^{-1/2}$. As can be seen from the form of the Gaussian approximation, Q-balls in the gravity mediated case have thick walls. Their charge-energy relation is well approximated by $E \approx mQ$.

1.1 Dissociation and Dissolution

A particle with energy T can penetrate a Q-ball up to a distance where its effective mass $g\phi(r)$ is equal to its energy *i.e.* $T \approx g\phi(r)$, where $\phi(r)$ is the Q-ball scalar field and g the coupling of the thermal particle to the ball. The thermal particle entering the ball will then lose some of its kinetic energy to the classical Q-ball field and come to a temporary halt. The excess energy transferred to the Q-ball can then be radiated as scalar field waves or lumps of charge, or some combination of these.

The rate of the number of particles hitting a Q-ball of radius R in a thermal

path of temperature T is

$$\frac{dN}{dt} = \frac{\tilde{g}(T)}{\pi^2} 4\pi R^2 T^3, \quad (2)$$

where $\tilde{g}(T)$ is the effective number of thermal degrees of freedom coupling to the Q-ball scalar. Typically we can set $\tilde{g}(T) \approx 10^2$.

The question of how much of the energy of an incoming particle will be transferred to the Q-ball was considered in [11]. Assuming that the Q-ball field can reconfigure itself over a time scale of m^{-1} , the Q-ball can typically absorb most of the energy of the incoming particle. Hence, in each collision an energy of $\gamma_T T$, where $\gamma_T \lesssim 1$, is transferred to the Q-ball. The rate at which energy is being added to the Q-ball by the thermal background is therefore given by

$$\frac{dE}{dt} = \gamma_T T \frac{dN}{dt} = \frac{\tilde{g}(T)}{\pi} 4\gamma_T R^2 T^4. \quad (3)$$

To avoid dissociation, a Q-ball should be able to radiate the excess energy as scalar field waves at a rate faster than given by (3). Otherwise, energy will be accumulated in the Q-ball until it begins to dissociate.

In addition to dissociation, a Q-ball can lose its charge by dissolution where charge is removed from the “soft edge” of the Q-ball. For $T \gtrsim m$, the thermal background quarks have a much smaller mean free path, $\lambda_{mfp} \approx k_q/T$ ($k_q \approx 6$) than the width of the soft edge of a Gaussian Q-ball, $\delta r \approx R/(2\beta)$, where $\beta = \ln(g\phi_0/3T)^{1/2}$. The charge contained in the soft edge is approximately $Q_{\text{soft}} \approx 4\pi\omega(T/g)^2(\beta R)^2\delta r$ [11]. The background quarks can therefore thermalize the soft edge of a Q-ball at high temperatures. The time taken for a charge to leave the soft edge by a random walk process is [11]

$$\tau_d \approx \left(\frac{R}{2\beta}\right)^2 \frac{T}{k_q} \approx \frac{T}{4\beta^2 m^2 |K| k_q}. \quad (4)$$

The rate at which charge can be removed then depends on how quickly the Q-ball can reconfigure itself and replenish charge in the soft edge. If the charge is replenished quickly enough, the rate of charge loss is $dQ/dt \approx Q_{\text{soft}}/\tau_d$, where Q_{soft} is the charge contained in the soft edge. However, if the Q-ball does not reconfigure and hence replenish the charge quickly enough, the rate of charge loss will be suppressed.

2 Numerical simulations

2.1 Preliminaries

To study the dynamics of excited Q-balls, they have been simulated numerically on a 2+1 dimensional lattice. Obviously, a 3+1 dimensional simulation would be more realistic but it is expected that a 2+1 dimensional simulation gives a fair approximation to the more realistic case while greatly reducing the required CPU time.

The equation of motion in flat space corresponding to the potential (1) is

$$\ddot{\Phi} + \nabla^2 \Phi + m^2 \Phi \left[1 + \frac{K}{\log(10)} + K \log\left(\frac{|\Phi|^2}{M^2}\right) \right] + \frac{2(d-1)\lambda^2}{M_{\text{Pl}}^{2(d-3)}} |\Phi|^{2(d-2)} \Phi = 0. \quad (5)$$

(We can neglect the expansion of the universe for our purposes since we are interested in the behaviour of a single, isolated Q-ball.)

Since our goal is to address questions relevant to Q-balls subjected to a thermal background radiation, it is necessary to consider exactly what to simulate while taking into account the numerical and practical problems that may arise.

Simulating the collision of a single particle of energy T with the Q-ball is cumbersome due to the vastly different energy scales involved. In a realistic cosmological setting, the energy of a Q-ball is typically much larger than T . Keeping track of such a tiny change in energy would require a simulation where the Q-ball field is let to evolve at very high accuracy while keeping the time and spatial steps very small. This in order requires a large lattice and several orders of magnitude more time steps. Also simulating a single particle hitting the Q-ball is not realistic since a Q-ball in the thermal bath of the early universe is subjected to several particles interacting with it simultaneously.

Another approach would be to fully simulate both a generic background field and the Q-ball field. The difficulty in this approach is to keep the background field at a constant temperature at large distances *i.e.* to keep the system from reaching a global equilibrium state due to the smallness of the lattice compared to the Q-ball size. By using a much larger lattice this approach would probably be more feasible but would again require more CPU time. Hence an approximate method has been adopted: instead of simulating the whole thermal erosion process, we simply study the evolution of a Q-ball with excess energy added in the form of a random perturbation, $\phi = \phi_0 + \delta\phi$, where ϕ_0 is the Q-ball field and $\delta\phi$ is a stochastic term. Such a perturbation adds to the energy of the Q-ball while keeping the contribution to the charge small. The exact form of the perturbation is not important; the relaxation times are insensitive of how the extra energy is added to the Q-ball. For example, if energy is added asymmetrically, it has been noted that the fluctuations of the field quickly distribute around the Q-ball before excess energy begins to radiate.

After the perturbation is added to the initial configuration, the field is allowed to develop until the Q-ball becomes stable again *i.e.* excess energy and possibly charge is radiated from the ball. To avoid any excess energy and possible charge radiated by the Q-ball from entering the system due to continuous boundary conditions, the lattice has dissipative zones near the boundaries (to minimize back-scattering from the boundaries of the dissipative regions the dissipation term is increased gradually). For the numerical simulations the field was decomposed into real and imaginary parts, $\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, and re-scaled along with the spatial and temporal unit, $\varphi_i = \frac{\phi_i}{\sqrt{2}M}$, $\tau = mt$, $\xi = mx$. The parameter values we used in the numerical simulations were $m = 100$ GeV, $K = -0.1$, $M = 10^9$ GeV. The calculations were done mainly for the $d = 4$ case but we have checked that the $d = 6$ case does not significantly differ from these results. We have also verified that varying K in the typical range $-(0.01 - 0.1)$ does not markedly affect the relaxation rate. The lattice size was typically 200×200 and the lattice parameters were chosen as $\Delta\xi = 0.6$, $\Delta\tau = 0.3$ (a number of simulations with varying values of $\Delta\xi$ and $\Delta\tau$ were done to ensure that the exact values of the lattice parameters do not affect the results).

2.2 Numerical results

We have studied numerically the relaxation of Q-balls for different Q-ball charges and excess energies. For each set of parameter values a number of simulations were run to account for the effect induced by the stochastic term *i.e.* a different seed for the pseudo random number generator in the stochastic term was used.

A typical relaxation process is shown in Fig. 1 where the amplitude of the scalar field is plotted. The initial configuration, with $\omega = 0.80m$ corresponding to $Q = 9100$ and $E = 7600$ (in lattice units), is shown in Fig. 1(a), is put on a 200×200 lattice and the field is let to evolve. Note that the images are not taken at equal time intervals: the time units at which snapshots are taken are $\tau_a = 0$, $\tau_b = 20$, $\tau_c = 40$, $\tau_d = 990$. From the figures it can be seen how the Q-ball quickly radiates the excess energy in the form of scalar field waves which propagate spherically from the ball. The waves are dissipated at the dissipation zones as they reach the boundaries of the lattice. Radiation gradually decreases and in the last panel one can see virtually no radiation. From Fig. 1(d) one can also note how the Q-ball has slightly moved from its initial position in the center of the lattice due to a spatial asymmetry present in the initial conditions.

The energy and charge of the Q-ball in lattice units have been plotted in Fig. 2. From the figure it can be seen how energy is emitted from the ball while the charge remains virtually constant, even though a significant amount of excess charge is introduced to the Q-ball initially. The slight fluctuation of charge at the beginning of the simulation is shown in the close up in Fig. 2(b).

To study how the energy is emitted from the Q-ball a number of different functions has been fitted to the energy curves. It is found that the energy curve is well described by

$$E(\tau) = A[\theta(\tau_0 - \tau) + \theta(\tau - \tau_0)e^{-\lambda(\tau - \tau_0)}] + E_0, \quad (6)$$

where A, λ, E_0 are the fitting parameters and $\theta(\tau)$ is the step function. The presence of a non-zero τ_0 -term implies that the excess energy first spreads around the ball before it is emitted.

The rate at which excess energy is emitted has been studied for $\omega = 0.78m - 0.98m$, which in two spatial dimensions corresponds to Q-ball charges in the range $Q \sim 10^{16} - 10^{18}$ (note that smaller values of ω correspond to larger charges). The values of λ have been plotted in Fig. 3 as a function of the unperturbed Q-ball energy (in lattice units). For each value of ω the maximum amplitude of the perturbation $\delta\phi$ was varied, $|\delta\phi| \leq a\phi(0)$, where a was varied in the range $0.01 - 0.10$. The maximum value of a for each ω was chosen so that the charge of the Q-ball varied less than 1% from its initial value during the relaxation process. Ten simulations were run with a different initial stochastic perturbation for each a .

The amount of excess charge is obviously a function of a and hence it varies greatly for each value of ω . For example, for the $\omega = 0.78m$ case, the values of λ have been calculated for increases in energy between 1% and 40% of the energy of the ground state. However, as from Fig. 3 can be seen, the variations in λ are small compared to the changes in energy. The values of λ increase slightly with smaller energy so that smaller Q-balls radiate their excess energy more quickly than larger Q-balls.

We have also studied the effect of adding larger amounts of excess energy to a Q-ball. The time development of energy and charge of a Q-ball with perturbations of different magnitude for $\omega = 0.80m$ is shown in Fig. 4. The ground state

has charge $Q_0 = 9100$ and energy $E_0 = 7600$. From the figure it can be seen how energy still decreases exponentially even with very large initial excess energy. As the amount of excess energy grows, the Q-ball emits more charge along with the energy but still retains most of its charge in a single Q-ball. In the case shown in Fig. 4, more than 75% of the initial charge remains in the Q-ball even if the excess energy is $\sim 10^5$. A Q-ball can hence survive fragmentation even with initial excess energies of the order of ~ 10 times its initial energy.

3 Discussion

From the numerical results we have seen that the energy loss experienced by an excited Q-ball is well described by an exponential function, $\dot{E}_Q(t) = -\lambda(E_Q(t) - E_{Q_0})$, where E_{Q_0} is the ground state of the Q-ball.

To study the dissociation of Q-balls in the early universe we need to consider the energy increase due to the thermal background. Since the rate at which energy is being added to the Q-ball due to the thermal background is given by (3), we can model the energy of the Q-ball in the early universe by

$$\dot{E}_Q(t) = -\lambda(E_Q(t) - E_{Q_0}) + \frac{\tilde{g}(T)}{\pi} 4\gamma_T R^2 T^4. \quad (7)$$

Here we can approximate that R is approximately independent of the charge (and energy) of the Q-ball, assuming that the energy increase is not large enough to dissociate the Q-ball.

The temperature, T , decreases with time due to the expansion the universe so that in a radiation dominated universe $T^2 t \sim M_{\text{Pl}}$ and in a matter dominated universe $T^{3/2} t \sim M_{\text{Pl}}$. In general one can therefore write (7) as

$$\dot{E}_Q(t) = -\lambda(E_Q(t) - E_{Q_0}) + \frac{A}{t^B}, \quad (8)$$

where A is a constant and $B = 2$ (radiation dominated) or $B = \frac{8}{3}$ (matter dominated). Integrating once with respect to time one gets

$$E_Q(t) = E_{Q_0} + (E_Q(t_0) - E_{Q_0})e^{-\lambda(t-t_0)} + A e^{-\lambda t} \int_{t_0}^t dx \frac{e^{\lambda x}}{x^B}, \quad (9)$$

where t_0 is now the time at which the Q-ball is formed.

The relevant features of the time evolution of Q-ball energy can be extracted from (7). As $t \rightarrow \infty$, the energy of a Q-ball tends to the ground state (assuming that the Q-ball does not dissociate during the process) *i.e.* $E_Q(t \rightarrow \infty) = E_{Q_0}$. If there exists an extremum t_c , $\dot{E}(t_c) = 0$, any subsequent extremum points, $\dot{E}(t) = 0$, will occur at a lower energy *i.e.* $E(t_c) > E(t)$ for $B > 1$. Hence the value of $E(t_0)$ determines the features of the time evolution of a Q-ball: If $E_Q(t_0) < A/(\lambda t_0^B) + E_{Q_0}$, $\dot{E}(t_0)$ is positive and the Q-ball energy will initially increase until the energy loss from the Q-ball exceeds the contribution from the background. The increase in energy is clearly bounded from above by $A t_0^{1-B}/(B-1)$. If, on the other hand, $\dot{E}(t_0) < 0$, the energy of a Q-ball will monotonously decrease with time.

If one assumes a situation where Q-balls are formed before reheating due to the inflaton decays and are approximately at their ground state, $E_Q(t = t_0) \approx$

E_{Q_0} , the energy of a Q-ball will initially increase, $\dot{E}(t) > 0$. The temperature of the universe after reheating can be approximated by $T(t) = \sqrt{\frac{M_{\text{Pl}}}{t}}$, $t \geq t_{\text{RH}} = M_{\text{Pl}}/T_{\text{RH}}^2$, where T_{RH} is the reheating temperature and t_{RH} is the reheating time. The maximum energy of a Q-ball clearly depends on the exact value of parameters. However, for typical parameter values, numerical work shows that for $T_{\text{RH}} \sim 10^2 - 10^9$ GeV the increase in energy is well approximated by

$$E_Q^{\text{max}}(t > t_{\text{RH}}) \approx E_Q(t_{\text{RH}}) + \frac{\tilde{g}(T)}{\pi\lambda} 4\gamma_T R^2 T_{\text{RH}}^4, \quad (10)$$

which corresponds to the point at which the rates at which energy is emitted and absorbed are equal. Clearly, (10) is valid only if the Q-balls emit their excess charge quickly enough compared to the rate of energy increase due to the energetic background.

From the simulations we know that to significantly remove charge from a Q-ball we need to add a perturbation whose energy is comparable to the energy of the Q-ball itself. Therefore, to avoid significant loss of charge the energy of a Q-ball is bounded from below by

$$E_Q \approx mQ \gtrsim \frac{\tilde{g}(T)}{\pi\lambda\alpha} 4\gamma_T R^2 T_{\text{RH}}^4, \quad (11)$$

where α is the fraction of excess energy that is required to significantly remove charge from the Q-ball. Using $R \sim m^{-1}|K|^{-1/2}$, $K = -0.1$, $\gamma_T \approx 1$ (*i.e.* all of the energy of an incoming particle is transferred to the Q-ball), $\tilde{g}(T) \approx 100$, $\lambda \sim 0.02m$ and $\alpha \sim 1$ we get

$$T_{\text{RH}} \lesssim 6 \times Q^{1/4} \text{ GeV}. \quad (12)$$

On the other hand, if Q-balls are still at an excited state at the time of reheating and the reheating temperature is low, the rate of energy loss from a Q-ball can be greater than the rate of energy increase due to reheating. The energy of a Q-ball can then be monotonously decreasing with time during the reheating process and dissociation due to the energetic background will not play a major role in the evolution of Q-balls.

In addition to dissociation, dissolution can also be a significant factor in determining the fate of Q-balls in the thermal bath of the early universe. For dissolution to be important the rate of charge loss from the soft edge, τ_d^{-1} , must be smaller than the rate at which charge is replenished. The rate which a Q-ball can reconfigure itself and replenish charge in the soft edge now becomes important. The numerical simulations show that the Q-ball emits excess charge at a slow rate, $\Gamma \sim \lambda m$ compared to dynamical scale of the scalar field, m . One can then speculate that there is a suppression factor in how quickly charge is replenished in the soft edge, $\tau_r \sim (\kappa m)^{-1}$.

For efficient dissolution we need $\tau_d^{-1} \lesssim \tau_r^{-1}$ so that using (4) we get a lower bound for the background temperature for ,

$$T \gtrsim 4\beta^2 m |K| k_q \kappa^{-1}. \quad (13)$$

Using typical values, we get $T \gtrsim \frac{4}{\kappa} \times 10^3$ GeV so that the presence of a suppression factor can significantly increase this bound. For example, assuming that $\kappa \sim \lambda$, we get $T \gtrsim 10^5$ GeV.

At temperatures high enough for efficient dissolution, charge in the soft edge is replenished quickly enough and dissolution is efficient. The rate of charge loss is then given by,

$$\frac{dQ}{dt} \approx \frac{Q_{\text{soft}}}{\tau_d} = 8\pi\omega\beta^3 k_q R g^{-2} T, \quad (14)$$

where (4) has been used. Using $T = (M_{\text{Pl}}/2k_T t)^{1/2}$ and integrating, the total loss of charge from the time of reheating until temperature has decreased to T , is given by

$$\Delta Q = 8\pi\omega\beta^3 k_q R g^{-2} k_T^{-1} M_{\text{Pl}} \left(\frac{1}{T} - \frac{1}{T_{RH}} \right). \quad (15)$$

The temperature at which the loss of charge due to dissolution is no longer efficient is given by (13) and hence (15) is bounded from above by

$$\Delta Q \leq \frac{2\pi\omega\beta\kappa M_{\text{Pl}}}{g^2 k_T m^2 |K|^{3/2}}. \quad (16)$$

For typical values $\omega \sim m$, $\beta \sim 4$, $g \sim 1$, $k_T \approx 17$ and $K = -0.1$ we get $\Delta Q \sim 0.5\kappa(M_{\text{Pl}}/\text{GeV})$. One should also note that the rate at which charge leaves the soft edge of a Q-ball by random walk will in reality be less than τ_d^{-1} since random walk transports charges in all spatial directions and the rate at which charge drifts away from the Q-ball will depend on the gradient of the charge density. Furthermore, charge diffusion away from the edge of the Q-ball needs to be efficient to prevent charges being transported back into the Q-ball. These considerations can further suppress the dissolution process.

4 Conclusions

We have studied the effect of adding excess energy to a Q-ball by numerical means. From doing a set of simulations we have found that the excess energy is lost as $E(t) \sim e^{-\lambda t} \sim e^{-\lambda m t}$, where $\lambda \sim 0.02 - 0.06$ in the studied parameter range. From naive arguments one would expect that the relaxation rate of the Q-ball would be of the order of m . The numerical simulations show that there is a suppression factor $\mathcal{O}(10^{-2})$ in how quickly Q-balls radiate their excess energy. We expect that this suppression factor would also be present in full 3D simulations even though the Q-balls are likely to radiate their excess energy slightly more quickly due to the presence of an additional degree of freedom. We have also observed the slow relaxation rate of excited Q-balls when studying Q-ball collisions [14].

In addition to the fact that Q-balls tend to radiate their excess energy more slowly than what one might expect, it has also been found that the Q-balls are surprisingly robust. Even when the excess energy added is a significant fraction of the energy of the unperturbed Q-ball and clearly exceeds the binding energy, $mQ - E$, only a very small fraction of the charge is lost in emitting the excess energy. Only when the excess energy is much larger than the Q-ball energy we start to see a significant loss of charge. One can gain an insight into this by looking at the effective potential of a single “free” ϕ particle inside the Q-ball: the particle is typically in a deep potential well with a high barrier that also suppresses quantum tunneling.

On the basis of the small rate of radiation of the excess energy, one can speculate that also the reconfiguration rate may be suppressed. This would

then reduce the rate at which a Q-ball can absorb energy from the thermal background and suppress the charge loss by dissolution. Especially the effect on dissolution is significant since a suppression factor in the reconfiguration rate affects linearly the lower bound for efficient dissolution (13) and hence also the upper bound for charge loss (16).

The slow relaxation rate and the robustness of Q-balls have an effect on how well they can avoid thermal erosion in the early universe. The slow relaxation rate allows for the excess energy to build up in a Q-ball but on the other hand the excess energy required to significantly reduce the charge of Q-ball by dissociation is large. In a realistic cosmological setting one should also take into account the effect of preheating due to inflaton decays and the decays of the remains of the AD-condensate. However, on the basis of the arguments presented in [11], preheating effects should not significantly affect the bounds for the Q-ball charge and reheat temperature.

Q-balls formed from the Affleck-Dine condensate can survive the thermal background of the early universe if the reheat temperature is not too large. On the basis of the results presented in this work Q-balls with charges $\sim 10^{20}$ should survive reheat temperatures around $T_{\text{RH}} \sim 10^5$ GeV. The question of how quickly a Q-ball can absorb energy and replenish charge in its soft edge can, however, change these limits so further work is still needed to understand the exact dynamics of how Q-balls erode thermally.

Acknowledgments

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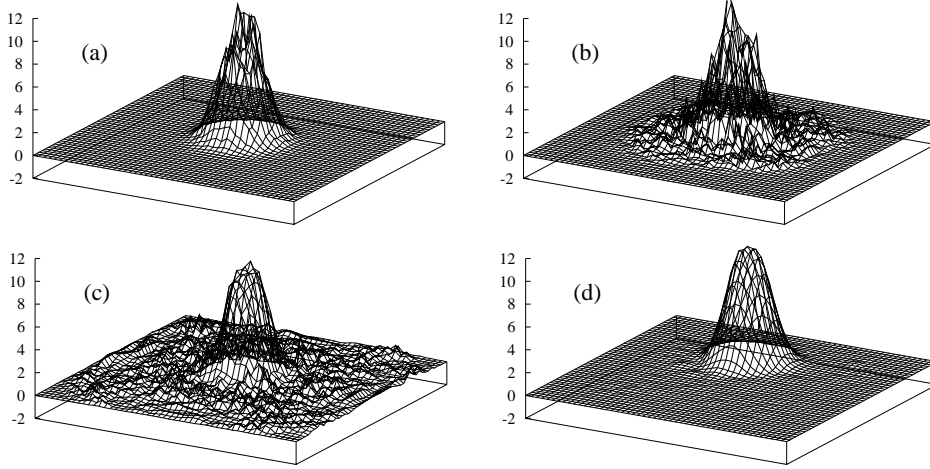


Figure 1: A typical Q-ball relaxation process

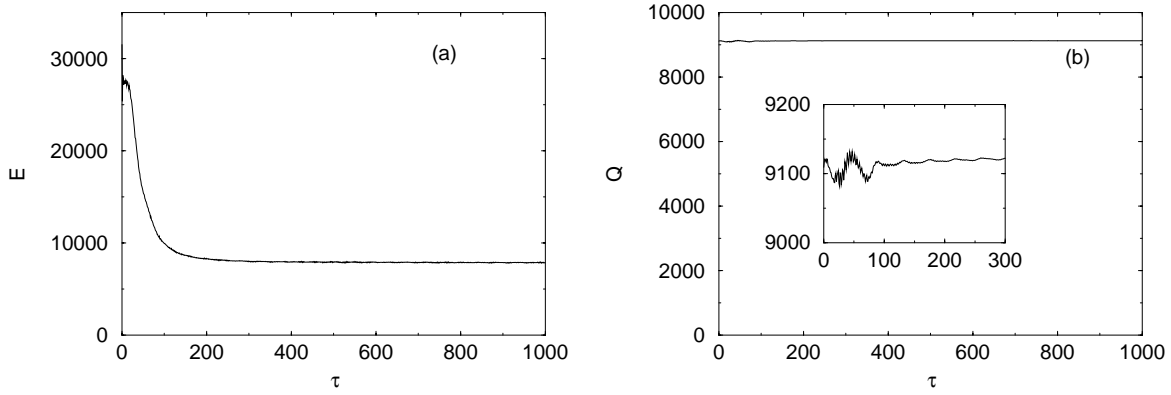


Figure 2: Q-ball energy (a) and charge (b) as a function of τ , $\omega = 0.80m$

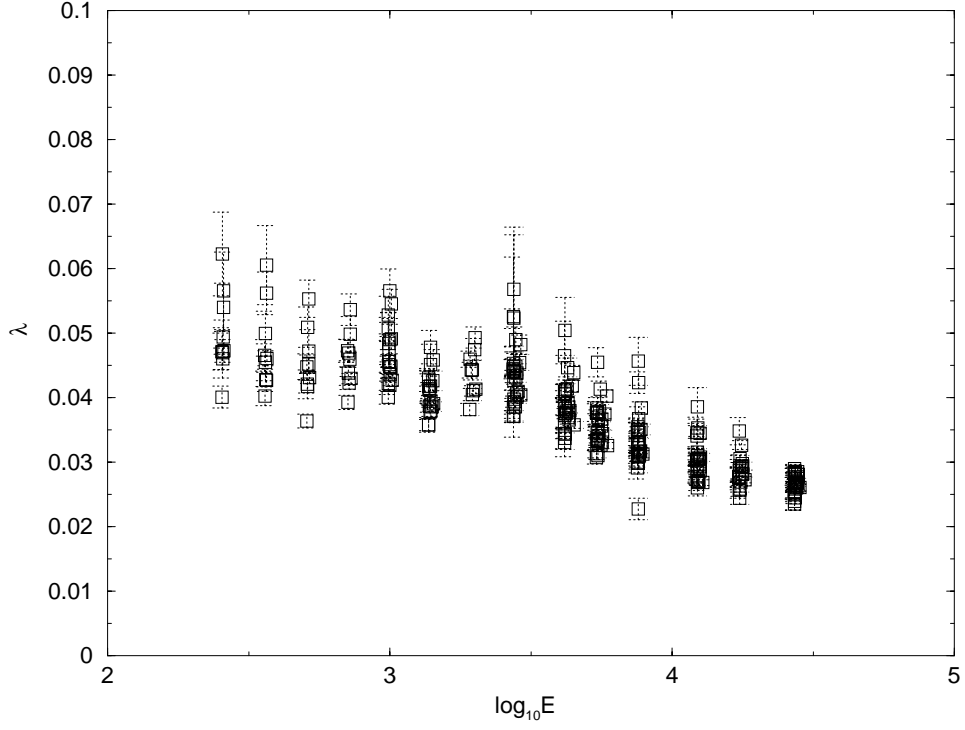


Figure 3: λ for $\omega = 0.78m - 0.98m$ as a function of Q-ball energy

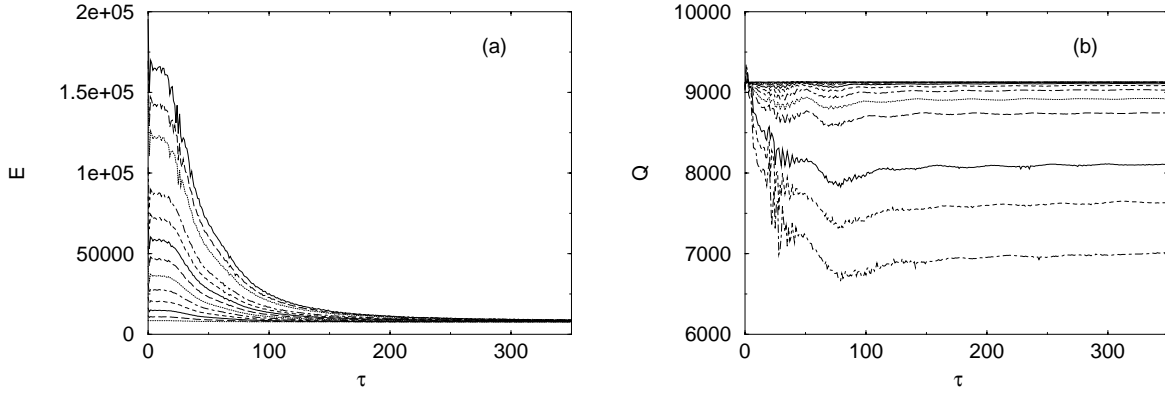


Figure 4: Q-ball energy (a) and charge (b), $\omega = 0.80m$, $Q_0 = 9100$, $E_0 = 7600$